MEM 637, Nonlinear Control Project 2

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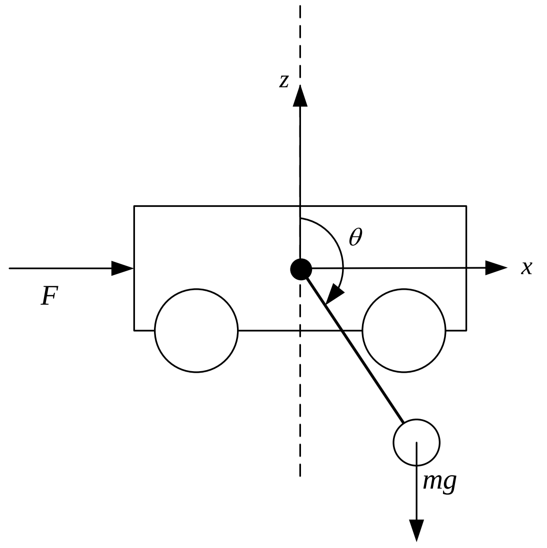
Consider the cart-pendulum system shown below. The force F, represents the net force imposed on the cart via the wheel motor. It is the primary control input and it is bounded which limits the acceleration. The goal is to swing the pendulum and bring it to an upright, stabilized, position.



The equations of motion are

$$\begin{bmatrix} \dot{x} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} v \\ \omega \end{bmatrix}$$

$$\begin{bmatrix} M_c + m_p & l m_p \cos\theta \\ l m_p \cos\theta & l^2 m_p \end{bmatrix} \begin{bmatrix} \dot{v} \\ \dot{\omega} \end{bmatrix} - \begin{bmatrix} F + l m_p \omega^2 \sin\theta \\ g l m_p \sin\theta \end{bmatrix} = 0$$

With normalized parameters

$$\begin{aligned} &\dot{x} = v \\ &\dot{\theta} = \omega \\ &2\dot{v} + \cos\theta\,\dot{\omega} = F + \omega^2 \sin\theta \\ &\cos\theta\,\dot{v} + \dot{\omega} = \sin\theta \end{aligned}$$

Now, rewrite F in terms of $\dot{v}$, i.e., $F = \left(2 - \cos^2\theta\right)\dot{v} - \left(\omega^2 - \cos\theta\right)\sin\theta$, where ω, θ are measured so that specification of $\dot{v}$ fully determines F. Thus, with the control input defined as $u = \dot{v}$, with constraint $|u| \le 1$, the control problem is reduced to the system

$\dot{\theta} = \omega, \quad \dot{\omega} = \sin\theta - u\cos\theta, \quad |u| \le 1$, Note that $E_{pend} = \tfrac{1}{2}\omega^2 + \left(\cos\theta - 1\right), \quad E_{pend} = 0 \text{ when } \omega = 0, \theta = 0$

The goal is to design a hybrid control swing-up strategy with three stages:

1. “pump/remove energy” into the system until $E_{pend} \approx 0,\ E_{pend} \in \left[-\varepsilon, \varepsilon\right]$,
   Note: $\dot{E} = \omega\left(\sin\theta - u\cos\theta\right) - \sin\theta\,\omega = -u\omega\cos\theta$
2. “Wait” until the pendulum is close to the upright equilibrium, $|\omega| + |\theta| < \delta$
3. “Engage” a feedback linearizing stabilizing controller.